DESCRIPTION OF PARTICLE MIXING IN A BOILING LAYER BY A HYPERBOLIC EQUATION

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It is shown that the horizontal diffusion coefficients of particles, which are found from the experimental response functions obtained using the parabolic and hyperbolic diffusion equations, are nearly identical.

It is known that the hyperbolic diffusion equation describes the nonstationary processes of particle mixing in a quasifluidized layer better than the classical (parabolic) equation [1]. If one takes into account the sufficiently large number of experimental data in the literature on the effective particle diffusion coefficients, found using the classical diffusion equation (see, e.g., [2, 3]), a question arises whether these coefficients can be used in the calculation of the nonstationary concentration and temperature fields from the hyperbolic equation.

Petrik and Taganov [4], in the analysis of experimental data on the vertical mixing of solid phase using parabolic and hyperbolic equations, obtained an unexpected and somewhat surprising result: The diffusion coefficients obtained using the hyperbolic equation considerably exceeded the corresponding diffusion coefficients found from the classical diffusion equation (e.g., in a free layer by a factor of 4, and in a retarded layer by a factor of 20-40!). This is even more surprising if we note that the diffusion coefficients determined in nonstationary conditions from the parabolic equation agree, as a rule, with the diffusion coefficients found from experiments according to standard methods [3, 5]. The reason for this substantial divergence of the coefficients determined in [4] from different models is possibly the neglect of circulation components of the particle velocities which lead to the "Taylor" diffusion, or in the incorrect form of the diffusion current of labeled impurities in the hyperbolic equation[$J_c = -D(\partial c/\partial x)$ instead of correct $J_c = -D(\partial c/\partial x) - \tau^*(\partial J_c/\partial\tau)$]

In the present problem we aim to study the applicability of using the effective horizontal diffusion coefficient of particles, obtained from the parabolic equation, for the description of nonstationary transport processes in structured boiling layers using the hyperbolic equation.

Borodulya et al. [6] studied the effective horizontal diffusion coefficients of particles (D_{ef}) in a layer retarded by a tube bundle. The experimental mixing curve was compared with the theoretical one:

$$\theta = \frac{t - t_0}{t_c - t_0} = \exp(-\mathrm{Pe}\,\mathrm{Fo})\left[R + 2\sum_{n=1}^{\infty}\frac{\sin n\pi R}{n\pi}\cos n\pi\xi\exp\left(-n^2\pi^2\,\mathrm{Fo}\right)\right], \quad (1)$$

which is the solution of the parabolic diffusion equation of heated particles (heat conduction equation),* with the corresponding boundary conditions

$$\frac{\partial\theta}{\partial\,\mathrm{Fo}} = \frac{\partial^2\theta}{\partial\xi^2} - \mathrm{Pe}\,\theta,\ \theta(0,\ \xi) = \begin{cases}1, & 0 \leqslant \xi < R,\\ 0, & R < \xi \leqslant 1,\end{cases}$$

*Noting the large difference between the volume heat capacities of the gas and particles, one can assume that all the heat in the system is transported by the heated moving particles.

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$$\frac{\partial\theta(\mathrm{Fo},\ 0)}{\partial\xi}=\frac{\partial\theta(\mathrm{Fo},\ 1)}{\partial\xi}=0. \tag{2}$$

The quantities D_{ef} were found from the condition for coincidence of the maxima of the experimental and calculated [Eq. (1)] curves.

The hyperbolic equation for the thermal horizontal particle diffusion with boundary conditions corresponding to (2) has the form

$$\frac{\partial\theta}{\partial\,\mathrm{Fo}}+\mathrm{Fo}^*\frac{\partial^2\theta}{\partial\,\mathrm{Fo}^2}=\frac{\partial^2\theta}{\partial\xi^2}-\mathrm{Pe}\,\theta\left(1+\mathrm{Fo}^*\frac{\partial}{\partial\,\mathrm{Fo}}\right), \tag{3}$$

$$\theta(0,\ \xi)=\begin{cases}1,\ 0\leqslant\xi<R,\\ 0,\ R<\xi\leqslant 1,\end{cases}$$

$$\frac{\partial\theta(0,\ \xi)}{\partial\,\mathrm{Fo}}+\mathrm{Pe}\,\theta(0,\ \xi)=0,\quad J=-\frac{\partial\theta}{\partial\xi}-\mathrm{Fo}^*\frac{\partial J}{\partial\,\mathrm{Fo}}=0,\quad \xi=0;\ \ 1. \tag{4}$$

To solve (3) and (4) we introduce a new function T by $\theta = \exp(-\mathrm{PeFo})T$. It is not difficult to show that the system (3), (4) reduces to a system of equations for T:

$$\frac{\partial T}{\partial\tilde{\mathrm{Fo}}}+\tilde{\mathrm{Fo}}^*\frac{\partial^2 T}{\partial\tilde{\mathrm{Fo}}^2}=\frac{\partial^2 T}{\partial\xi^2},$$

$$T(0,\ \xi)=\begin{cases}1,\ 0\leqslant\xi<R,\\ 0,\ R<\xi\leqslant 1,\end{cases}\quad\frac{\partial T(0,\ \xi)}{\partial\tilde{\mathrm{Fo}}}=0, \tag{5}$$

$$J_T=-\frac{\partial T}{\partial\xi}-\tilde{\mathrm{Fo}}^*\frac{\partial J_T}{\partial\tilde{\mathrm{Fo}}}=0\quad\text{for}\quad\xi=0;\ 1. \tag{6}$$

The new dimensionless complexes can be expressed as follows: $\tilde{\mathrm{Fo}}^* = \mathrm{Fo}^*/(1-\mathrm{PeFo}^*)^2$, $\tilde{\mathrm{Fo}} = \mathrm{Fo}/(1-\mathrm{PeFo}^*)$, $J_T = J\exp(\mathrm{PeFo})(1-\mathrm{PeFo}^*)$. The solution of the system (5), (6) can be obtained by the following method. Teplitskii [7] studied the circulation model of the vertical mixing of particles of Van Deemter [8]:

$$A\left(\frac{\partial c_1}{\partial\tau}+u_1\frac{\partial c_1}{\partial x}\right)=\beta(c_2-c_1),$$

$$B\left(\frac{\partial c_2}{\partial\tau}-u_2\frac{\partial c_2}{\partial x}\right)=\beta(c_1-c_2) \tag{7}$$

with boundary conditions

$$c_1(0,\ x)=c_2(0,\ x)=\begin{cases}c_c,\ 0\leqslant x<h,\\ c_0,\ h<x\leqslant H,\end{cases}$$

$$c_1(\tau,\ 0)=c_2(\tau,\ 0),\ c_1(\tau,\ H)=c_2(\tau,\ H). \tag{8}$$

The solution of (7), (8) was obtained in the form [7]

$$\theta_i=\frac{c_i-c_0}{c_c-c_0}=R+\exp(-J_0/\mathrm{Pe}^*)\sum_{n=1}^{\infty}\frac{(-1)^n\sin n\pi R}{n\pi}\left(\Gamma_i\,\mathrm{ch}\,\frac{J_0}{\mathrm{Pe}^*}A_n+\frac{D_i}{A_n}\,\mathrm{sh}\,\frac{J_0}{\mathrm{Pe}^*}A_n\right)\ (i=1,\ 2,\ 3), \tag{9}$$

where

$$\Gamma_1=\Gamma_2=\Gamma_3=2\cos n\pi(1-\eta);\ D_1=\Gamma_1-2\mathrm{Pe}^*\,n\pi\sin n\pi(1-\eta);$$

$$D_2=\Gamma_2+2\,\mathrm{Pe}^*\,n\pi\sin n\pi(1-\eta);\ D_3=\Gamma_3+2\frac{B-A}{A+B}\,\mathrm{Pe}^*\,n\pi\sin n\pi(1-\eta);\ A_n=\sqrt{1-\mathrm{Pe}^{*2}n^2\pi^2};$$

$$J_0 = \frac{A-B}{A+B}\left(-\eta + R + \frac{2\tau_0 \mathrm{Pe}^*}{A-B}\right);\quad \theta_3 = \frac{A\theta_1 + B\theta_2}{A+B}.$$

Eliminating in turn c_1 and c_2 from (7), we obtain for the average concentration $c_3 = c = (Ac_1 + Bc_2)/(A + B)$

$$\frac{\partial c}{\partial \tau} + \frac{AB}{\beta(A+B)}\frac{\partial^2 c}{\partial \tau^2} + \frac{AB(u_1-u_2)}{\beta(A+B)}\frac{\partial^2 c}{\partial \tau \partial x} = \frac{u^2}{\beta(A+B)}\frac{\partial^2 c}{\partial x^2}. \tag{10}$$

It is seen from Eq. (10) that the system (7) is a generalization of the hyperbolic diffusion equation of the form (5) if one postulates different velocities of the incident and reflected waves (u_1 and u_2, respectively). Substituting $u_1 = u_2$ into (10) and introducing the notation $AB/\beta(A + B) = \tau^*$, $u^2/\beta(A + B) = D^q_{ef}$ we obtain a hyperbolic diffusion equation in the usual form:

$$\frac{\partial c}{\partial \tau} + \tau^* \frac{\partial^2 c}{\partial \tau^2} = D^a_{ef}\frac{\partial^2 c}{\partial x^2}. \tag{11}$$

In dimensionless variables, Eq. (11) becomes

$$\frac{\partial \theta_3}{\partial \bar{\mathrm{Fo}}} + \bar{\mathrm{Fo}}^* \frac{\partial^2 \theta_3}{\partial \bar{\mathrm{Fo}}^2} = \frac{\partial^2 \theta_3}{\partial \eta^2}. \tag{12}$$

Using the equality $Au_1 = Bu_2 = u$, the boundary conditions in (8) give the absence of current of the labeled impurity in the cross sections x = 0 and x = H. Equation (11) (or the equivalent Eq. (12)) with boundary conditions (8) corresponds, except for the notation, to the system (5), (6) whose solution can therefore be obtained from (9) by putting here A = B, $u^2/\beta(A + B) = D^*_{ef}/(1 - \beta^*\tau^*)$, $AB/\beta(A + B) = \tau^*/(1 - \beta^*\tau^*)$.

The solution of the hyperbolic equation (5) with the boundary conditions (6) which is thus found has the form

$$T = R + 2\exp\left(-\frac{\tilde{\mathrm{Fo}}}{2\tilde{\mathrm{Fo}}^*}\right)\sum_{n=1}^{\infty}\frac{\sin n\pi R\cos n\pi\xi}{n\pi}\left[\mathrm{ch}\left(\frac{\tilde{\mathrm{Fo}}}{2\tilde{\mathrm{Fo}}^*}\sqrt{1-4n^2\pi^2\tilde{\mathrm{Fo}}^*}\right) + \frac{\mathrm{sh}\left(\frac{\tilde{\mathrm{Fo}}}{2\tilde{\mathrm{Fo}}^*}\sqrt{1-4n^2\pi^2\tilde{\mathrm{Fo}}^*}\right)}{\sqrt{1-4n^2\pi^2\tilde{\mathrm{Fo}}^*}}\right]. \tag{13}$$

Finally, the solution of the problem (3), (4) is

$$\theta = \exp(-\mathrm{Pe\,Fo})\left\{R + 2\exp\left(-\frac{\tilde{\mathrm{Fo}}}{2\tilde{\mathrm{Fo}}^*}\right)\sum_{n=1}^{\infty}\frac{\sin n\pi R\cos n\pi\xi}{n\pi}\times\right.$$
$$\left.\times\left[\mathrm{ch}\left(\frac{\tilde{\mathrm{Fo}}}{2\tilde{\mathrm{Fo}}^*}\sqrt{1-4n^2\pi^2\tilde{\mathrm{Fo}}^*}\right) + \frac{\mathrm{sh}\left(\frac{\tilde{\mathrm{Fo}}}{2\tilde{\mathrm{Fo}}^*}\sqrt{1-4n^2\pi^2\tilde{\mathrm{Fo}}^*}\right)}{\sqrt{1-4n^2\pi^2\tilde{\mathrm{Fo}}^*}}\right]\right\}. \tag{14}$$

It is not difficult to show that for Fo* = 0 (and accordingly $\tilde{\mathrm{Fo}} = \mathrm{Fo}$, $\tilde{\mathrm{Fo}}^* = 0$) Eq. (14) reduces to the solution of the parabolic diffusion equation (2), i.e., to formula (1).

We estimate the term $\mathrm{PeFo}^* = \beta^*\tau^*$ in the expressions for $\tilde{\mathrm{Fo}}$ and $\tilde{\mathrm{Fo}}^*$. Borodulya et al. [6] found β^* experimentally, using the theory of regular regime. The values of this quantity were found to lie in the range $\beta^* = 0.002$–0.009 1/sec; in retarded layers, $\tau^* = 0.2$–5 sec ([1], p. 172). Thus $\beta^*\tau^*$ varies within the limits 0.0004–0.05 and it is permissible to put $\tilde{\mathrm{Fo}}^* = \bar{\mathrm{Fo}}^*$, Fo = Fo in (14).

The experimental curves of the thermal horizontal particle diffusion obtained in [6] were compared with the results obtained on a computer according to (14) (Fig. 1). The condition of the best fit, which was measured by the sum of squares of the deviations of the calculated points from the experimental ones, gave the required values of D^*_{ef} and $\tau^* = D^*_{ef}/w^2$ (in Fig. 1a, $D^*_{ef} = 12$ cm^2/sec, and $\tau^* = 5.3$ sec). The velocity of the incident and reflected waves (w) was determined from the retardation time (τ_r) of the experimental response function (see Figs. 1 and 2; the experimental functions in Figs. 1a and 2 are identical): w = (y —

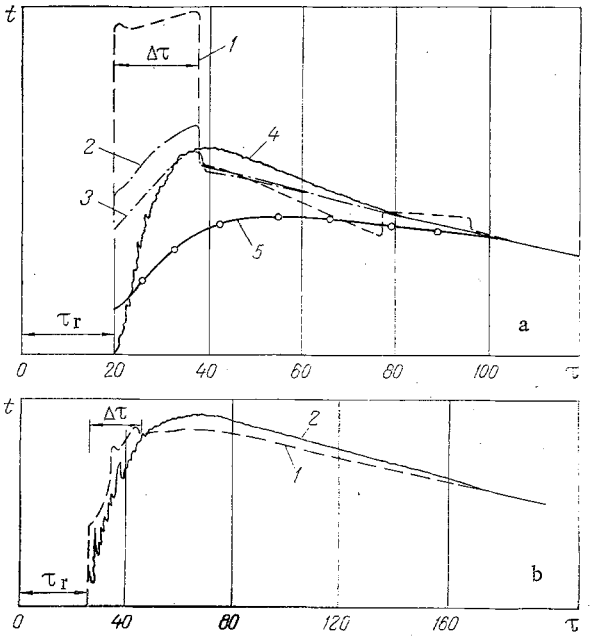

Fig. 1. The comparison of the experimental response functions with those calculated from the hyperbolic equation. a) $(u/\varepsilon_p) - u_o$ = 144 cm/sec; w = 1.5 cm/sec; β^* = 0.0075 1/sec; ξ = 0.85; D^*_{ef} = 12.0 cm^2/sec; τ^* = 5.3 sec; $\Delta\tau$ = 4h/w. Curve 1: D = 28.4 cm^2/sec, 2: D = 14.2 cm^2/sec, 3: D = 12.0 cm^2/sec, 5: D = 7.1 cm^2/sec. Curve 4 is the experimental function. b) $(u/\varepsilon_p) - u_o$ = 60 cm/sec; w = 1.15 cm/sec; β^* = 0.0039 1/sec; ξ = 0.90; τ^* = 5.6 sec; $\Delta\tau$ = 3.33 h/w. Curve 1: D^*_{ef} = 7.4 cm^2/sec; curve 2 is the experimental function. The quantity τ is in seconds.

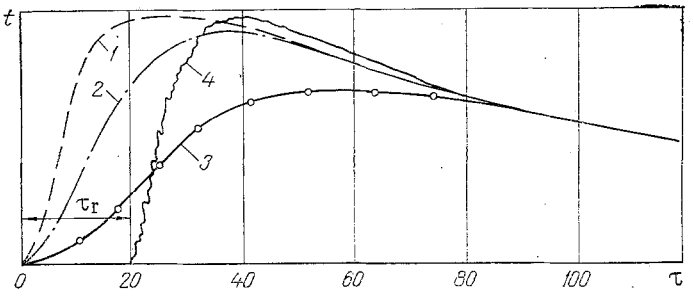

Fig. 2. The comparison of the experimental response function with the one calculated from the parabolic equation: $(u/\varepsilon p) - u_o$ = 144 cm/sec; w = 1.5 cm/sec; β^* = 0.0075 1/sec; ξ = 0.85; D_{ef} = 14.2 cm^2/sec. Curve 1: D = 28.4 cm^2/sec; 2: D = 14.2 cm^2/sec; 3: D = 7.1 cm^2/sec. Curve 4 is the experimental function.

h)/τ_r, where y is the coordinate of the point where the temperature was measured. For comparison, Fig. 2 shows the results of calculation of the response function using the solution of the parabolic diffusion equation (1). The maxima of curve 2 and of the experimental response function 4 coincide. As we noted above, this condition determines the required diffusion coefficient D_{ef} from the parabolic model (in Fig. 2, D_{ef} = 14.2 cm^2/sec). It is seen

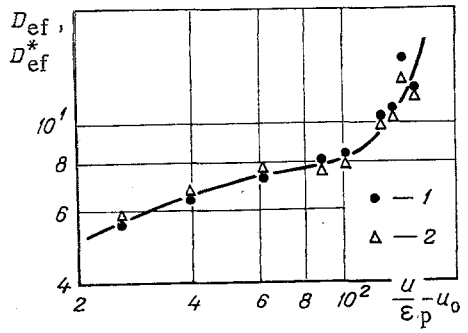

Fig. 3. The horizontal particle diffusion coefficients D_{ef} and D^*_{ef} (cm^2/sec) as functions of the air velocity $u/\varepsilon_n - u_0$ (cm/sec). Curve 1: D_{ef}; curve 2: D^*_{ef}.

that the hyperbolic diffusion equation describes the experimental dependences (Fig. 1) considerably better than the parabolic equation (Fig. 2). For the concrete experiment (Figs. 1a and 2) the diffusion coefficient determined from the hyperbolic equation (D^*_{ef}) is close to the diffusion coefficient found from the parabolic equation (D_{ef}). In an analogous fashion we analyzed all experimental curves of thermal diffusion obtained in [6]. The results are shown in Fig. 3. They indicate that the coefficients D_{ef} and D^*_{ef} are nearly identical.

Thus, although the problem of applicability of diffusion coefficients obtained from the parabolic equation for the calculation of temperature and concentration profiles using the hyperbolic equation requires a further study, the presented results indicate that this is possible, at least within the limits of the experimental range studied in the present work.

NOTATION

A, volume fraction of the layer taken up by the descending continuous phase; A_n, number introduced in (9); B, volume fraction of the layer taken up by the ascending continuous phase (relative volume of bubble trails); c_1, c_2, mass of the labeled substance per unit volume of the descending continuous phase, and of the hydrodynamic bubble trails, respectively; $c = c_3 = (Ac_1 + Bc_2)/(A+B)$; Γ_i, D_i (i = 1, 2, 3), coefficients in (9); D_{ef}, horizontal particle diffusion coefficient (the coefficient of horizontal thermal diffusivity of the layer) found from the parabolic equation (2); D^*_{ef}, horizontal diffusion coefficient found from the hyperbolic equation (3); $D^a_{ef} = u^2/\beta(A + B)$, axial "Taylor" diffusion coefficient defined in (10); H, height of the layer; h, width of the initial rectangular pulse; J, dimensionless heat flux; l, width of the layer; $R = h/H$, vertical miscibility; $R = h/l$, horizontal miscibility; t, t_0, t_c, temperature of the layer, initial temperature, and temperature of the heating chamber, respectively; u, u_0, filtration and initial fluidization velocities, respectively; u_1, velocity of the descending continuous phase; u_2, velocity of the bubble trail; $Au_1 = Bu_2 = u$, circulation velocity of the particles taking into account the total cross section of the apparatus; w, wave velocity; x, y, vertical and horizontal coordinates; β^* characterizes the intensity of the heat dissipation from the layer; β, volume of the solid particles and of the gas between them exchanged per unit time per unit volume of the layer between the descending continuous phase and the bubble trail; ε_p, fraction of volume of the layer not taken up in packing; $\eta = x/H$; $\theta = (t - t_0)/(t_c - t_0)$; $\xi = y/l$; $\tau_0 = \beta\tau$; τ, time; τ^*, relaxation time; τ_r, retardation time; $Fo = D_{ef}\tau/l^2$ (for the parabolic equation); $Fo = D^*_{ef}\tau/l^2$ (for the hyperbolic equation); $\overline{Fo} = D^a_{ef}\tau/H^2$; $Fo^* = D_{ef}\tau^*/l^2$; $\tilde{Fo}^* = D^a_{ef}\tau^*/H^2$; $\tilde{Fo} = Fo/(1 - PeFo^*)$; $\tilde{Fo}^* = Fo^*/(1 - PeFo^*)^2$; $Pe^* = u/\beta H$; $Pe = \beta^* l^2/D_{ef}$ (for the parabolic equation); $Pe = \beta^* l^2/D^*_{ef}$ (for the hyperbolic

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PARTICLE VELOCITY DISTRIBUTION IN THE FLOW OF AQUEOUS POLYETHYLENE SOLUTIONS IN A CIRCULAR PIPE

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A distribution of longitudinal velocity, averaged over the cross section in the turbulent flow of water and aqueous solutions of polyethylene in thin pipes, is obtained on a nuclear magnetic resonance unit.

A distribution function for longitudinal velocity pulsations was obtained in [1] in the turbulent flow of water and aqueous solutions of polyethylene in a pipe. Unfortunately, the experiment was not sensitive to the sign of the pulsations, and the function obtained was averaged relative to positive and negative pulsations.

For a new experiment, we used the same nuclear magnetic resonance (NMR) unit as in [1], with slight differences (Fig. 1). The liquid flowing through the pipe, 700 mm long with a 6.0 mm inside diameter (1), was magnetized positively by polarizer 2. By means of the IMI-2 magnetic induction meter, we used detector 3 to record an NMR signal of an intensity proportional to the magnetization M of a unit volume of the flowing liquid. The change in the NMR signal over time was photographed from the screen of an S8-7A memorizing oscillograph. The nutation coil 5 was powered by a G4-26 generator. The liquid was pumped from a 50-liter container by pump 6. The pump was located outside of the pipe to avoid degradation of the polyethylene solution.

Particles of the liquid were marked by the nutation coil, which created a weak variable magnetic field directed perpendicular to the vector of the external magnetic field in the coil. The G4-26 generator created a variable electrical field with a frequency of 100 kHz — equal to the nucleus precession frequency — and a power corresponding to a 180° rotation of

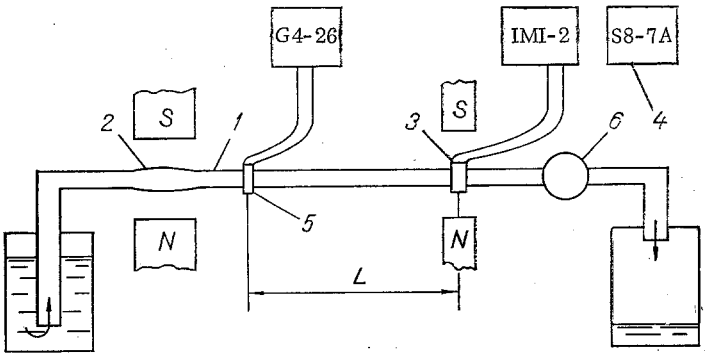

Fig. 1. Block diagram of experimental unit for obtaining a particle velocity distribution function in liquid flow in a pipe.

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